These distributions are used for continuous RVs to describe random processes in time.

Random Process : Something that happens 'randomly' at a rate $\lambda$.

→ Therefore in a small period of time, $\Delta t$

- the probability of one event is ~ $\lambda \Delta t$, probability of no event $1 - \lambda \Delta t$
- probability of more than one event very small (scales like $\Delta t^2$)

Deriving Exponential Distribution :

Suppose event happens at time $t=0$, what's the pdf $f(t)$ for the time of the next event?

- Cumulative distribution $F(t)$ is the prob. that the next event happens before or at time $t$.

↳ let $G(t) = 1 - F(t)$ be the probability the event hasn't happened before time $t$

- If we have a small time interval $\Delta t$, then

$$G(t+\Delta t) = G(t)(1-\lambda t)$$

the probability it didn't happen in time $t$

the probability it didn't happen in $\Delta t$

So $$G(t+\Delta t) - G(t) = -G(t)\lambda\Delta t$$

$$\frac{G(t+\Delta t) - G(t)}{\Delta t} = -G(t)\lambda \longrightarrow G'(t) = -\lambda G(t)$$

so $$G(t) = \exp(-\lambda t)$$

as when you take derivative it gives some function times a constant

so $$\boxed{F(t) = 1 - \exp(-\lambda t)}$$

and $$\boxed{f(t) = F'(t) = \lambda \exp(-\lambda t)}$$

$\lambda$ is the mean rate per unit time at which some event occurs.

$\boxed{\text{Mean time} = \frac{1}{\lambda}}$ , $\boxed{\text{Variance} = \frac{1}{\lambda^2}}$

If you take the continuous time scale and split it into a finite number of intervals, $n\Delta t$, you can use Poisson :

Associating continuous process with discrete process.

$1\Delta t \quad 2\Delta t \quad 3\Delta t \quad \dots n\Delta t \quad T$

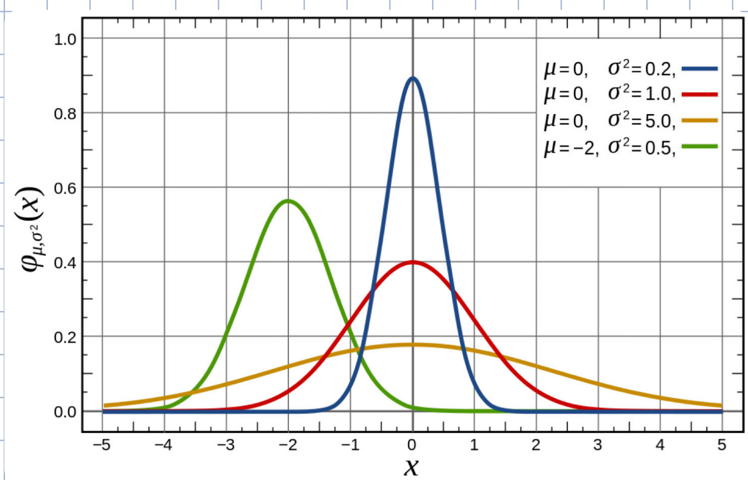
Let $Y$ be discrete RV describing the number of events in time interval $T$.

$Y$ is the Poisson parameter $\lambda T$

$$P_Y(T) = P(Y=k) = \frac{\lambda T^k}{k!} e^{-\lambda T}$$

'probability of $k$ events within finite interval of time $T$.'

## Normal Distribution :




mean, $\mu$, shifts curve

standard deviation, $\sigma$, determines spread (eg. peak lower, wider spread)

$$\int_{-\infty}^{\infty} f_X(x)\,dx = 1$$

$\therefore$ if peak lower, curve wider as area $\equiv 1$

$$F_X(x) = \boxed{\frac{1}{\sigma\sqrt{2\pi}} \boxed{\int_{-\infty}^{x} \exp\left[-\frac{1}{2\sigma^2}(x-\mu)^2\right] dx}}$$ cdf

pdf

For any Normally Distributed Variable :

68.3% of values lie between $\mu - \sigma$ and $\mu + \sigma$

95.45% between $\mu \pm 2\sigma$

99.73% between $\mu \pm 3\sigma$

## Converting to Standard Normal Distribution :

$$X \sim N(\mu, \sigma^2)$$

'If $X$ is a normally distributed variable with mean $\mu$ & variance $\sigma^2$'

then $$Z = \frac{X - \mu}{\sigma} \sim N(0,1)$$

## Normal Distribution Table :

- Shows $P(z \leq c)$ so if you have $P(z \geq \bar{c}) = x$ you must do $1 - P(z \leq \bar{c}) = 1 - x$

## Central Limit Theorem :

Let $X$ be a RV with mean $\mu$ and variance $\sigma^2$

If you consider large number of trials

$$Y = X_1 + X_2 + X_3 \ldots + X_n$$

then $$Y \sim N(n\mu, n\sigma^2)$$

'Sum is normally distributed with mean of $n\mu$ and variance $n\sigma^2$.'